6.4 Prove Triangles Similar by AA

Before You used the AAS Congruence Theorem.

Now You will use the AA Similarity Postulate.

Why? So you can use similar triangles to understand aerial photography, as in Ex. 34.

Key Vocabulary

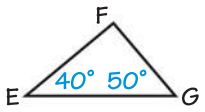
- **similar polygons,** *p. 372*

ACTIVITY Angles and Similar Triangles

QUESTION What can you conclude about two triangles if you know two pairs of corresponding angles are congruent?

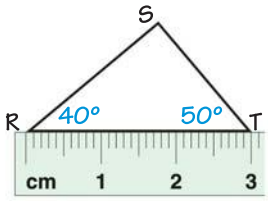
Materials:
- protractor
- metric ruler

STEP 1 **Draw** $\triangle EFG$ so that $m\angle E = 40°$ and $m\angle G = 50°$.



STEP 2 **Draw** $\triangle RST$ so that $m\angle R = 40°$ and $m\angle T = 50°$, and $\triangle RST$ is not congruent to $\triangle EFG$.

STEP 3 **Calculate** $m\angle F$ and $m\angle S$ using the Triangle Sum Theorem. Use a protractor to check that your results are true.



STEP 4 **Measure** and record the side lengths of both triangles. Use a metric ruler.

DRAW CONCLUSIONS

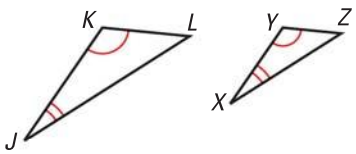
1. Are the triangles similar? Explain your reasoning.
2. Repeat the steps above using different angle measures. Make a conjecture about two triangles with two pairs of congruent corresponding angles.

TRIANGLE SIMILARITY The Activity suggests that two triangles are similar if two pairs of corresponding angles are congruent. In other words, you do not need to know the measures of the sides or the third pair of angles.

POSTULATE *For Your Notebook*

POSTULATE 22 Angle-Angle (AA) Similarity Postulate

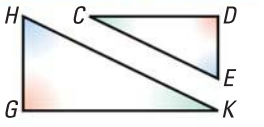
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.



$\triangle JKL \sim \triangle XYZ$

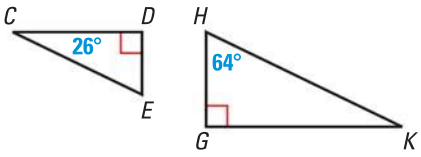
EXAMPLE 1 Use the AA Similarity Postulate

DRAW DIAGRAMS



Use colored pencils to show congruent angles. This will help you write similarity statements.

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.



Solution

Because they are both right angles, $\angle D$ and $\angle G$ are congruent.

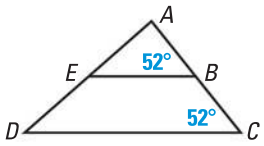
By the Triangle Sum Theorem, $26° + 90° + m\angle E = 180°$, so $m\angle E = 64°$. Therefore, $\angle E$ and $\angle H$ are congruent.

▶ So, $\triangle CDE \sim \triangle KGH$ by the AA Similarity Postulate.

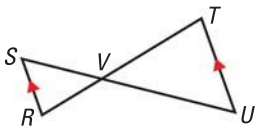
EXAMPLE 2 Show that triangles are similar

Show that the two triangles are similar.

a. $\triangle ABE$ and $\triangle ACD$



b. $\triangle SVR$ and $\triangle UVT$



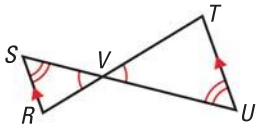
Solution

a. You may find it helpful to redraw the triangles separately.

Because $m\angle ABE$ and $m\angle C$ both equal 52°, $\angle ABE \cong \angle C$. By the Reflexive Property, $\angle A \cong \angle A$.

▶ So, $\triangle ABE \sim \triangle ACD$ by the AA Similarity Postulate.

b. You know $\angle SVR \cong \angle UVT$ by the Vertical Angles Congruence Theorem. The diagram shows $\overline{RS} \parallel \overline{UT}$ so $\angle S \cong \angle U$ by the Alternate Interior Angles Theorem.

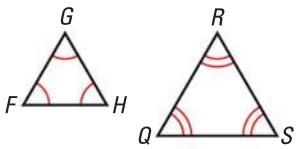


▶ So, $\triangle SVR \sim \triangle UVT$ by the AA Similarity Postulate.

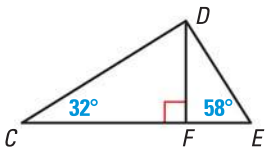
✓ GUIDED PRACTICE for Examples 1 and 2

Show that the triangles are similar. Write a similarity statement.

1. $\triangle FGH$ and $\triangle RQS$



2. $\triangle CDF$ and $\triangle DEF$



3. REASONING Suppose in Example 2, part (b), $\overline{SR} \not\parallel \overline{TU}$. Could the triangles still be similar? *Explain.*

INDIRECT MEASUREMENT In Lesson 4.6, you learned a way to use congruent triangles to find measurements indirectly. Another useful way to find measurements indirectly is by using similar triangles.

EXAMPLE 3 Standardized Test Practice

A flagpole casts a shadow that is 50 feet long. At the same time, a woman standing nearby who is five feet four inches tall casts a shadow that is 40 inches long. How tall is the flagpole to the nearest foot?

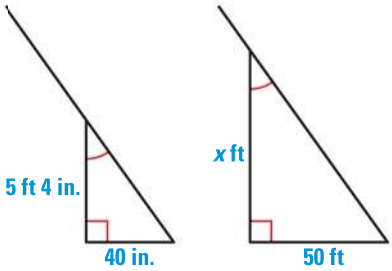
Ⓐ 12 feet Ⓑ 40 feet

Ⓒ 80 feet Ⓓ 140 feet

ELIMINATE CHOICES
Notice that the woman's height is greater than her shadow's length. So the flagpole must be taller than its shadow's length. Eliminate choices A and B.

Solution

The flagpole and the woman form sides of two right triangles with the ground, as shown below. The sun's rays hit the flagpole and the woman at the same angle. You have two pairs of congruent angles, so the triangles are similar by the AA Similarity Postulate.



You can use a proportion to find the height x. Write 5 feet 4 inches as 64 inches so that you can form two ratios of feet to inches.

$\frac{x \text{ ft}}{64 \text{ in.}} = \frac{50 \text{ ft}}{40 \text{ in.}}$ **Write proportion of side lengths.**

$40x = 64(50)$ **Cross Products Property**

$x = 80$ **Solve for x.**

▶ The flagpole is 80 feet tall. The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

GUIDED PRACTICE for Example 3

4. **WHAT IF?** A child who is 58 inches tall is standing next to the woman in Example 3. How long is the child's shadow?

5. You are standing in your backyard, and you measure the lengths of the shadows cast by both you and a tree. Write a proportion showing how you could find the height of the tree.

6.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 13, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 18, 19, 20, 33, and 38

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are ? .

2. ★ **WRITING** Can you assume that corresponding sides and corresponding angles of any two similar triangles are congruent? *Explain*.

EXAMPLE 1 on p. 382 for Exs. 3–11

REASONING **Use the diagram to complete the statement.**

3. $\triangle ABC \sim$?

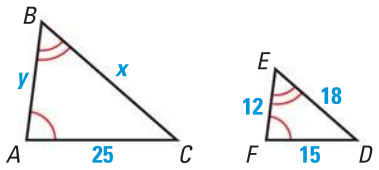
4. $\frac{BA}{?} = \frac{AC}{?} = \frac{CB}{?}$

5. $\frac{25}{?} = \frac{?}{12}$

6. $\frac{?}{25} = \frac{18}{?}$

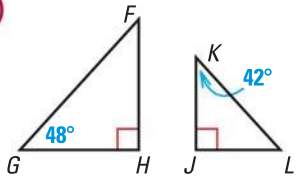
7. $y =$?

8. $x =$?

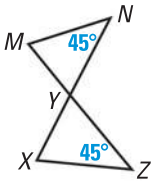


AA SIMILARITY POSTULATE **In Exercises 9–14, determine whether the triangles are similar. If they are, write a similarity statement.**

9.



10.



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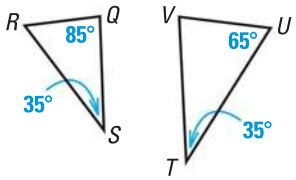
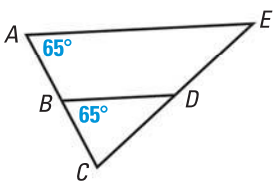
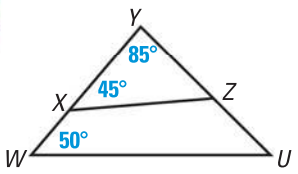


EXAMPLE 2 on p. 382 for Exs. 12–16

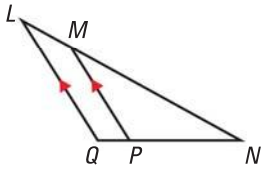
12.



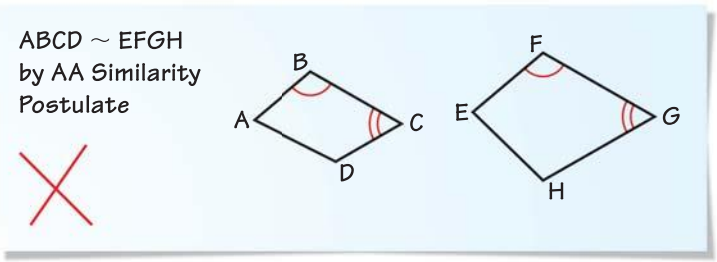
13.



14.



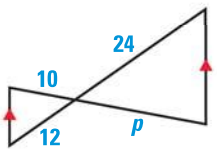
15. **ERROR ANALYSIS** *Explain* why the student's similarity statement is incorrect.



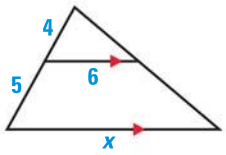
16. ★ **MULTIPLE CHOICE** What is the value of p?

Ⓐ 5 Ⓑ 20

Ⓒ 28.8 Ⓓ Cannot be determined



17. ERROR ANALYSIS A student uses the proportion $\frac{4}{6} = \frac{5}{x}$ to find the value of x in the figure. *Explain* why this proportion is incorrect and write a correct proportion.



★ OPEN-ENDED MATH In Exercises 18 and 19, make a sketch that can be used to show that the statement is false.

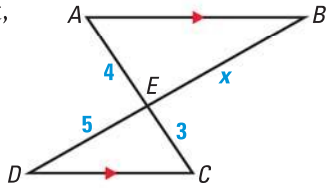
18. If two pairs of sides of two triangles are congruent, then the triangles are similar.

19. If the ratios of two pairs of sides of two triangles are proportional, then the triangles are similar.

20. ★ MULTIPLE CHOICE In the figure at the right, find the length of $\overline{BD}$.

Ⓐ $\frac{35}{3}$ Ⓑ $\frac{37}{5}$

Ⓒ $\frac{20}{3}$ Ⓓ $\frac{12}{5}$



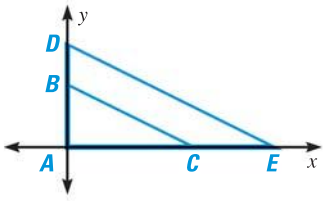
xy ALGEBRA Find coordinates for point E so that $\triangle ABC \sim \triangle ADE$.

21. $A(0, 0)$, $B(0, 4)$, $C(8, 0)$, $D(0, 5)$, $E(x, y)$

22. $A(0, 0)$, $B(0, 3)$, $C(4, 0)$, $D(0, 7)$, $E(x, y)$

23. $A(0, 0)$, $B(0, 1)$, $C(6, 0)$, $D(0, 4)$, $E(x, y)$

24. $A(0, 0)$, $B(0, 6)$, $C(3, 0)$, $D(0, 9)$, $E(x, y)$



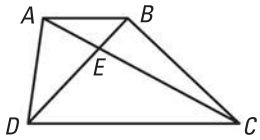
25. MULTI-STEP PROBLEM In the diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$, $AE = 6$, $AB = 8$, $CE = 15$, and $DE = 10$.

a. Copy the diagram and mark all given information.

b. List two pairs of congruent angles in the diagram.

c. Name a pair of similar triangles and write a similarity statement.

d. Find BE and DC.



REASONING In Exercises 26–29, is it possible for $\triangle JKL$ and $\triangle XYZ$ to be similar? *Explain* why or why not.

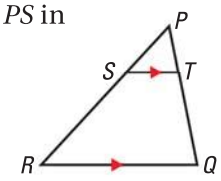
26. $m\angle J = 71°$, $m\angle K = 52°$, $m\angle X = 71°$, and $m\angle Z = 57°$

27. $\triangle JKL$ is a right triangle and $m\angle X + m\angle Y = 150°$.

28. $m\angle J = 87°$ and $m\angle Y = 94°$

29. $m\angle J + m\angle K = 85°$ and $m\angle Y + m\angle Z = 80°$

30. CHALLENGE If $PT = x$, $PQ = 3x$, and $SR = \frac{8}{3}x$, find PS in terms of x. *Explain* your reasoning.

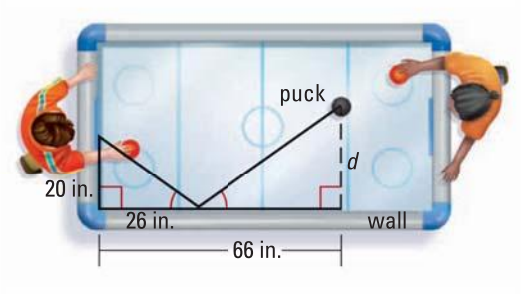


PROBLEM SOLVING

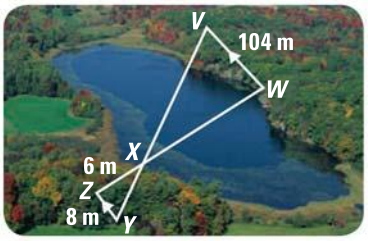
EXAMPLE 3 on p. 383 for Exs. 31–32

31. AIR HOCKEY An air hockey player returns the puck to his opponent by bouncing the puck off the wall of the table as shown. From physics, the angles that the path of the puck makes with the wall are congruent. What is the distance d between the puck and the wall when the opponent returns it?

@HomeTutor for problem solving help at classzone.com



32. LAKES You can measure the width of the lake using a surveying technique, as shown in the diagram.



a. What postulate or theorem can you use to show that the triangles are similar?

b. Find the width of the lake, WX.

c. If $XY = 10$ meters, find VX.

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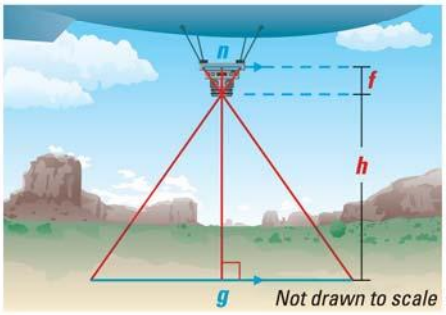
33. ★ SHORT RESPONSE *Explain* why all equilateral triangles are similar. Include sketches in your answer.

34. AERIAL PHOTOGRAPHY Low-level aerial photos can be taken using a remote-controlled camera suspended from a blimp. You want to take an aerial photo that covers a ground distance g of 50 meters. Use the proportion $\frac{f}{h} = \frac{n}{g}$ to estimate the altitude h that the blimp should fly at to take the photo. In the proportion, use $f = 8$ centimeters and $n = 3$ centimeters. These two variables are determined by the type of camera used.



35. PROOF Use the given information to draw a sketch. Then write a proof.

GIVEN ▶ $\triangle STU \sim \triangle PQR$
Point V lies on $\overline{TU}$ so that $\overline{SV}$ bisects $\angle TSU$.
Point N lies on $\overline{QR}$ so that $\overline{PN}$ bisects $\angle QPR$.

PROVE ▶ $\frac{SV}{PN} = \frac{ST}{PQ}$

36. PROOF Prove that if an acute angle in one right triangle is congruent to an acute angle in another right triangle, then the triangles are similar.

○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

37. **TECHNOLOGY** Use a graphing calculator or computer.

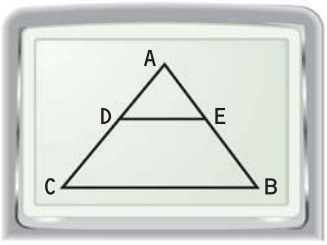
a. Draw $\triangle ABC$. Draw $\overline{DE}$ through two sides of the triangle, parallel to the third side.

b. Measure $\angle ADE$ and $\angle ACB$. Measure $\angle AED$ and $\angle ABC$. What do you notice?

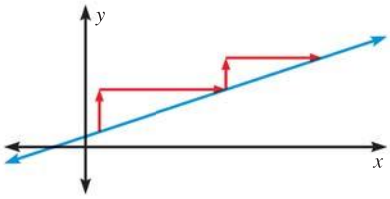
c. What does a postulate in this lesson tell you about $\triangle ADE$ and $\triangle ACB$?

d. Measure all the sides. Show that corresponding side lengths are proportional.

e. Move vertex A to form new triangles. How do your measurements in parts (b) and (d) change? Are the new triangles still similar? *Explain*.



38. ★ **EXTENDED RESPONSE** *Explain* how you could use similar triangles to show that any two points on a line can be used to calculate its slope.



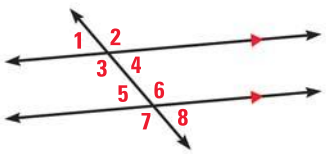
39. **CORRESPONDING LENGTHS** Without using the Corresponding Lengths Property on page 375, prove that the ratio of two corresponding angle bisectors in similar triangles is equal to the scale factor.

40. **CHALLENGE** Prove that if the lengths of two sides of a triangle are a and b respectively, then the lengths of the corresponding altitudes to those sides are in the ratio $\frac{b}{a}$.

MIXED REVIEW

PREVIEW
Prepare for Lesson 6.5 in Exs. 41–44.

In Exercises 41–44, use the diagram.



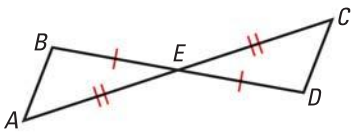
41. Name three pairs of corresponding angles. *(p. 147)*

42. Name two pairs of alternate interior angles. *(p. 147)*

43. Name two pairs of alternate exterior angles. *(p. 147)*

44. Find $m\angle 1 + m\angle 7$. *(p. 154)*

45. **CONGRUENCE** Explain why $\triangle ABE \cong \triangle CDE$. *(p. 240)*



Simplify the ratio. *(p. 356)*

46. $\frac{4}{20}$ **47.** $\frac{36}{18}$ **48.** $21:63$ **49.** $42:28$